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# Stochastic uncertainty modelling for ship design loads and operational guidance

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ABSTRACT

Understanding and quantification of uncertainties are important attributes for the assessment of the performance, reliability and risk of complex engineering structures and systems. From a naval architecture perspective the consideration of the uncertainties related to ship's seakeeping responses and wave induced loads is necessary for the assessment of ship design as well as for safe and efficient ship operations. To this end, the efficient processing of large amounts of data and the stochastic load combination by use of available numerical models may assist with the rationalisation of modelling assumptions and support the validation and verification of design and decision making criteria in the context of risk based ship design. This paper presents recent advances in (a) modelling the combined hydrodynamic responses of ship structures using cross-spectral combination methods and (b) in implementing uncertainty models used for the development of modern decision support systems as guidance to ship's master.



## 1. Introduction

Principal objective of ship design and operation is to ensure that the safety and performance requirements of the asset are best met for the required period of operational life. This is inherently performed under conditions of uncertainty due to the stochastic nature of the marine environment. Modelling uncertainties may vary and be related to the limitations of various assumptions made in the formulations of sea spectra, estimations of seakeeping responses, hydrodynamic loads as well as to the choice of a ship's master to decide and perform safely under random and, sometimes, extreme environmental conditions.

Within the area of ship wave loading and risk adverse ship performance it is well recognised that the difficulty in the determination of the wave-induced seakeeping responses and associated design loads depends on the understanding, modelling and validation of their global and local effects on the hull girder (ITTC, 2011a; Hirdaris et al., 2012). From a practical perspective, depending on the engineering problem specifics and within the context of maritime safety, uncertainties are specified by reliability-based code formats (e.g. Bitner-Gregersen et al. 2002) and navigation guidance concepts (e.g. Spanos et al., 2008; Bitner-Gregersen and Skjong, 2009). For example, suitable fluid-structure idealisation of the hull dynamics (e.g. ship flexibility, bow flare, slenderness, forward speed) and their random distributions in time and space using advanced stochastic load combination techniques is useful for ship design assurance. On the other hand in recent years with the advent of performance based standards there has also been increasing focus on the application of onboard, real-time guidance for ships as well as for offshore structures in terms of decision support systems (DSS). The latter are studied, developed and applied in a wide range of contexts aiming to reduce fuel consumption and to increase the operational and navigational safety of ships for improved safety with regard to ship-to-ship operations, and within the context of modelling of risk adverse ship traffic prioritisation (Nielsen et al., 2011).

The subject of modelling and implementing uncertainties in maritime technology related sub-themes is vast and it is not the purpose of this paper to identify firm solutions or categorise approaches that may be used for holistic engineering assessment (e.g. combination of mathematical, experimental or numerical means). The authors, in an attempt to emphasise the importance of understanding and integrating uncertainties in the context of useful tools for ship design and operations, elaborate on some indicative examples of relevant recent research work. The latter refers to : (a) the importance of developing cross-spectral stochastic mathematical methods for the combined hydrodynamic response of ship structures and (b) the implementation of uncertainty models in decision support systems (DSS) as guidance to the ship's master.

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## 2. Review of recent advances

### 2.1. General terms of reference

In a broad sense uncertainties can be categorised into two groups namely (a) aleatory (natural and physical) and (b) epistemic or knowledge based (Hirdaris et al., in press). Accordingly, information about uncertainties may be introduced in a reliability analysis by random variables. Aleatory uncertainties represent the natural randomness of a quantity, also known as an intrinsic or inherent uncertainty, e.g. the variability in wave intensity over time, which cannot be reduced or eliminated. Epistemic uncertainties represent errors which can be reduced by collecting more information about a considered quantity and by improving the methods of measuring it. These uncertainties may be classified into (a) data related, (b) statistics related and (c) model related. Data uncertainties appear due to the imperfection of an instrument used to measure a quantity, and/or a model generating data. If a considered quantity is not obtained directly from the measurements, but some estimation process is interposed (e.g. the significant wave height), then the measurement of uncertainty must be combined with the estimation of model uncertainty by appropriate means. Statistical uncertainty, often also referred to as estimation uncertainty, is due to limited information such as a limited number of observations of a quantity and also due to an estimation technique applied for the evaluation of the distribution parameters. Model uncertainty is due to imperfections, simplifications and idealisations made in physical model formulations for an event as well as in choices of probability distribution types used in the representation of uncertainties. The accuracy of a quantity characterises the extent to which a measured quantity agrees with the true value. Hence, indicating a systematic error (also known as bias) and a precision (or random) error is critical. The systematic error, or bias, of an estimator for a quantity considered refers to a systematic deviation from the true value of the quantity. The precision of the quantity considered refers to random variations, and is usually summarised by the standard deviation. A normal distribution is commonly adopted to describe the precision.

In recent years, the International Towing Tank Conference (e.g. ITTC, 2011b) has greatly contributed in rationalising the use of aerospace and ISO-GUM uncertainty standards for the benefit of the maritime technology/scientific community. The ISO-GUM ITTC proposed uncertainty concept, although not completely unified, distinguishes between two types of standard uncertainty components, namely Types A and B. Whereas these types are not considered substitutes for random and systematic uncertainties used under AIAA (1999, 2003), they are grouped according to the way their numerical values are estimated. Their derivation is based on the following five principles:

- *Principle 1:* Type A uncertainties are evaluated by applying statistical methods to the results of a series of repeated measurements. Type B uncertainties are those evaluated by other means than the use of statistical methods.
- *Principle 2:* The components in Type A uncertainty are defined by the estimated variance, $s_i^2$ or standard deviation $s_i$ of the mean value and number of degrees of freedom $\nu_i$, which includes the effect of the number of degrees of freedom.
- *Principle 3:* The components in Type B uncertainty are also approximated by a corresponding variance, in which its existence is assumed. However, Type B components are characterised by the quantity $u_j^2$ that can be thought of as the variance or standard deviation $u_j$ obtained based on a pool of reliable information from past experience and educated judgment.
- *Principle 4:* The combined uncertainty should be computed by the normal method for the combination of variances, now known as the law of propagation of uncertainty.
- *Principle 5:* For particular applications, the combined uncertainty should be multiplied by a coverage factor to obtain an overall uncertainty value. The overall uncertainty is now called expanded uncertainty. For the 95% confidence level, the coverage factor is 2.

When a final experimental outcome is obtained from results of few individual quantities, the ultimate uncertainty is called combined standard uncertainty $u_C(y)$. The uncertainty of the measured result can be then evaluated using relevant variance values and applying the law of propagation of uncertainty, namely

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \tag{1}$$

In addition, if there is a correlation between individual input quantities the combined standard uncertainty needs to be corrected by an expression correlating these quantities, namely

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) \tag{2}$$

When an interval about the result of a measurement to express an overall uncertainty, and associated variability and specific level of confidence in the final results is required the expanded uncertainty $U$ has to be used. This is obtained by multiplying the combined standard uncertainty by the so-called coverage factor $k$, i.e.

$$U = k u_C(y) \tag{3}$$

The value of $k$ is selected based on the required level of confidence. Normally in engineering applications 95% or 99% confidence level is acceptable. The relevant $k$ values for normal distribution are 2 and 3 respectively.

### 2.2. Uncertainties in wave load predictions – recent advances

Uncertainty in wave loads may be divided into uncertainty of wave loads calculated under linearity assumptions by linear hydrodynamic models and uncertainty related to employed non-linear hydrodynamic models. Uncertainties of linear wave load predictions refer to the shape of wave spectra, the choice of wave scatter diagram, linear transfer functions and to methods for the prediction of long-term extreme values and human actions. Uncertainty of non-linear models in wave loads prediction may comprise of different sagging and hogging bending moments, mainly due to non-vertical ship sides, and the influence of slamming and whipping on the extreme vertical bending moments.

The traditional approach for assessing the wave-induced loads on intact ship structures assumes that the sea states are dominated by wave systems generated by local winds. However, in many situations marine structures are subjected to the simultaneous action of more than one wave systems and in this case the frequency spectrum exhibits two peaks. Double-peaked wave spectra can be observed when a swell system combines with wind-generated waves. For example, Teixeira and Guedes Soares (2009) have demonstrated that, for a trading ship of non-restricted operation, the long-term distributions of the wave-induced vertical bending moment for combined sea states do not change significantly when compared with the ones obtained from sea states of a simple component. In their work it is recognised that double-peaked wave spectra can have a significant impact on the

design and operability of fixed and offshore platforms. They suggest that it would be important to assess separately damaged ships, since collisions and groundings may occur in sea areas with swell dominated sea states and the manoeuvrability may be affected as a consequence of the accident.

Ivanov (2009) proposed a method for calculating the hull girder bending stresses following the procedure specified in the classification rules but in probabilistic terms. In this work the still water and wave-induced hull girder hogging and sagging loads are presented in probabilistic format as one phenomenon, i.e. using bi-modal probability density functions. The probabilistic distribution of the total hull girder load is calculated using the rules of the composition of the distribution laws of the constituent variables. Parunov and Ćorak (2010) investigated the influence of environmental and operational uncertainties on the long-term extreme vertical wave bending moment of a containership assuming rigid hull. As the long-term distributions of vertical wave bending moments are highly dependent on the assumed environmental and operational parameters, their different combinations are considered. Results are compared among themselves as well as with the IACS rule vertical wave bending moments. Statistical parameters which may be useful for reliability-based design of containerships are quantified. Shu and Moan (2008) studied the effect of the heavy weather avoidance on the long-term wave-induced pressure along the mid-ship transverse section of a VLCC and a bulk carrier. They proposed a practical model to consider the effect of heavy weather avoidance on the wave pressure along a mid-ship transverse section.

Jensen (2009) provided a discussion of useful stochastic procedures for wave load problems covering the range from slightly linear to strongly non-linear (bifurcation) problems. For the methods employed, namely (a) hermite transformation, (b) critical wave episodes and (c) first-order reliability method (FORM) procedures are illustrated by results for the extreme vertical wave bending moment in ships. Another simplified procedure for determining the long-term distribution of wave hull girder loads acting on containerships including transient loads such as slamming and green water effects is presented by Jensen et al. (2009). The authors combined high frequency transient loads with lower frequency wave-induced loads, whereas the entire simplified solution is presented in a closed format solution. It is shown that for non-linear processes a good estimate for the mean out-crossing rate can be found using FORM. In the same work a notable, but not general, property of the FORM analysis for the specific problem of the extreme bending moment is identified; that is the associated reliability index is inversely proportional to the significant wave height for fixed values of other operational parameters. As pointed out by Jensen (2010) this means that the computational efficiency of Monte Carlo simulations for the specific problem can be increased drastically by introducing a scaling of the significant wave height. Gaidai et al. (2010) described a method for the prediction of extreme whipping stresses measured on deck amidships a container vessel during operation in harsh weather. Whipping response time series were analysed for two different voyages of the same ship, similar route and similar season month. Two different statistical methods were applied and compared with respect to the extreme response estimate. Parunov et al. (2011) investigated long-term distribution of slamming loads of containerships accounting for different types of environmental and operational uncertainties. In this work the uncertainties studied were (a) the choice of the wave scatter diagram, (b) the effect of the avoidance of heavy weather, (c) the effect of the manoeuvring in heavy weather and (d) the method for predicting the long-term extreme slamming pressures.

### *2.3. Uncertainties and decision support systems – the risk based perspective*

Typically the underlying approach for operational guidance builds on combined theoretical seakeeping models and linear spectral analysis used for statistical predictions. Recently, concepts of novel procedures for operational guidance have been proposed to increase the reliability. Many new-built ships have extensive data collection systems that are used for continuous monitoring of the engine and hull performance, for voyage performance and evaluation, etc. Often, such systems are, or could be, expanded to include also procedures for operational guidance, where statistics of the most critical wave-induced ship extreme responses and fatigue damage accumulation can be estimated for hypothetical changes in ship course and speed. The focus on goal-based standards – e.g. Papanikolaou et al. (2010), Papanikolaou (2009), and Skjong and Guedes Soares (2008) – implies that future developments of operator guidance systems should be based on numerical models that introduce probabilistic and risk-based approaches. Further remarks and discussions about risk-based methods for operational guidance have also been outlined by Shigunov et al. (2010), Bitner-Gregersen and Skjong (2009) and Nielsen et al. (2009).

The current state of the art in operational guidance typically relies upon mathematical models in which the on-site wave environment is automatically estimated. Ongoing developments in EU FP7 WATERBORNE project Handling Waves (2011) are driven by the development of systems that could be used for monitoring in real time the actual ship responses and associated structural loads due to weather changes and to possible changes in course or speed. This research programme supports the notion that the calibration of load predictions and the development of simplified numerical models that are accurate and fast are necessary in order to ensure that information and guidance are given with sufficient time to the ship's master. This is, for example, in further investigated and demonstrated in the work by Nielsen et al. (2009) and Nielsen and Stredulinksy (2010). In these investigations the horizontal acceleration and the racking failure mode of containers stowed on ships in heavy weather are studied. A procedure which can be used to obtain up-crossing rates for an inherent non-linear ship response, such as the racking force in containers, is derived. It is also shown that first- and second-order reliability-based formulations (FORM/SORM) and associated procedures may be significantly faster than more crude simulations (e.g. Monte Carlo). The motion simulation of container stacks on deck is considered also by Wolf and Rathje (2009). The authors deal with a (refined) numerical model from which knowledge about the dynamic forces acting on container stacks can be attained. The numerical findings and results of this work could be useful for establishing decision support criteria with respect to container and lashing loads. Considerations of computational efficiency in relation to calculation of fatigue damage rates in the ship hull girder and operator guidance have been presented by Ito et al. (2010) and Nielsen et al. (2011).

Typically, the underlying approach for operational guidance builds on a pure mathematical model only, where seakeeping characteristics of the ship, often given in terms of response amplitude operators (RAOs), are combined with information about the on-site sea state using linear spectral analysis to make statistical predictions of future responses to be expected. However, the on-site estimation of sea state parameters at the location of an advancing ship forms a crucial and fundamental problem to which a perfect solution has not been found yet. For this reason, concepts of a novel procedure for operational guidance have been proposed by Nielsen and Stredulinksy (2010) and Nielsen and Jensen (2011). The purpose of the procedure presented is to increase the reliability of the given guidance. Thus, predictions of future response levels are based on an integrated model using a mathematical model that has as input the estimated sea state parameters, and using also past measurements of the considered

response(s). Both works include an analysis of full-scale motion measurements, and the approach shows promising results.

A number of studies have recently been dedicated particularly towards onboard monitoring of fatigue damage rates. Models and procedures have been developed to evaluate fatigue damage accumulation in the hull girder both for short-term (30 min–3 h) decision support (e.g. Nielsen et al., 2011) and for long-term voyage planning (e.g. Mao et al., 2010). These studies present comparisons between measurements and predictions of fatigue damage rates and promising results are obtained. As uncertainties related to fatigue damage analysis can be profound independently of the prediction period, it should be considered to use risk-based approaches for the evaluation of fatigue damage rates. This has not been attempted until now, but ideas may be gained from the studies by Mao et al. (2010) and Choung et al. (2010).

## 3. Global wave load combinations by cross-spectral methods

One of the uncertainty modelling related difficulties in the determination of the wave-induced design loads and corresponding effects is that the loads are randomly distributed in time and space and even within the context of a stochastic process their individual design extreme values occur at different times. As suggested by Ayyub et al. (2000) whereas a probabilistic approach may be used to establish the basis for a reliability-based ship design methodology, the later demands that load effects and associated combinations have to be determined apriori. Various deterministic and stochastic methods for load combinations have been proposed in the literature by e.g. Ayyub et al. (2000), Ferry-Borges and Castenheta (1971), and Sun and Bai (2003). Comparison of different load combination solutions by Guedes Soares (1992) and Wang and Moan (1996) demonstrated that in those cases where the load combination problem is approached as a virtual superposition of stochastic load processes the maxima of individual load processes are assumed to be smaller than the sum of the combined load peak magnitudes, while load combinations are dependent on the associated models.

There has been no proven suitable adaptable method for all types of load models as even spectral analysis techniques are constraint by the identification of independent signal processes. In theory, this gap could be covered via utilisation of the so-called cross-spectral analysis techniques. Those, claim to reduce uncertainty via allowing for the determination of the relationship between two time series as a function of frequency. In such approaches periodicities may be related with each other and their phase relationship is defined via determining the 'cross-spectrum', a multifunctional variable that defines the magnitude of the in-phase and 90° out-of-phase relationship between processes. An additional advantage is that such methods can be extended in the form of a higher order spectral analysis to account for non-linear wave load associations.

In an attempt to shed some light in the usefulness and applicability of advanced reliability-based methodologies for the assessment of combined loads on ships and offshore structures, the following sections present key results from the cross-spectral stochastic methodology developed by Alfred Mohammed et al. (2012). In this approach the hydrodynamic load combination takes into account uncertainties associated with the joint probabilities of the wave period and height, ship heading, speed, etc., is implemented via a cross-spectral analysis of the frequency domain hydrodynamic loads in conjunction with bi-variant probability density functions or the co-variances of two random variables with their associated derivatives. The bi-variate Rayleigh model is used because it provides a Copula function (Nelsen, 2006). That is a function that has parameters which are obtainable from the spectra of the random variables whose combination is sought. Hence it reduces uncertainty with regard to the overall derivation of short-term or long-term combined wave load predictions.

### 3.1. Theoretical approach

The detailed specifics of the cross-spectral approach are reported in Alfred Mohammed et al. (2012). The following sections outline only key points of the mathematical background.

#### 3.1.1. Short-term load combination

If we take $x$ and $y$ to denote two random variables which are narrow-banded Gaussian processes with zero means and variances $m_{0x}$ and $m_{0y}$ respectively and the correlation between them is $\rho_{xy}$. The peaks of their respective excursions can be modelled by Rayleigh density functions given by the following equation:

$$f_x(x)=\frac{x}{m_{0x}}e^{(-x^2/2m_{0x})} \quad \text{and} \quad f_y(y)=\frac{y}{m_{0y}}e^{(-y^2/2m_{0y})} \tag{4}$$

The joint probability density function of the peaks of these processes can be expressed by the bi-variate Raleigh probability density function of the random variables $x$ and $y$ as

$$f_{xy}(x,y)=\frac{xy}{m_{0x}m_{0y}(1-\rho_{xy}^2)}e^{\left\{(-1/(1-\rho_{xy}^2))\times((x^2/2m_{0x})+(y^2/2m_{0y}))\right\}} \times I_0\left(\frac{\rho_{xy}xy}{(1-\rho_{xy}^2)m_{0x}m_{0y}}\right) \tag{5}$$

where

$$I_0(z)=\sum_{k=0}^{\infty}\frac{\left(\frac{z}{2}\right)^{2k}}{(k!)^2} \tag{6}$$

$$\rho_{xy}=\frac{E(xy)}{\sqrt{E(x^2)E(y^2)}} \tag{7}$$

$$\begin{aligned} E(x^2)&=\int_0^{\infty}|H_x(\omega)|^2S(\omega)\,d\omega \\ E(y^2)&=\int_0^{\infty}|H_y(\omega)|^2S(\omega)\,d\omega \\ E(xy)&=\int_0^{\infty}\frac{1}{2}Re\left[H_x(\omega)H_y^*(\omega)+H_y(\omega)H_x^*(\omega)\right]S(\omega)\,d\omega \end{aligned} \tag{8}$$

In the above equations, $S(\omega)$ is the wave spectrum, $H_x(\omega)$ and $H_y(\omega)$ are the response amplitude operators (RAOs) of $X$ and $Y$ respectively; $H_x^*(\omega)$ and $H_y^*(\omega)$ are their respective conjugates; $E$ is the mathematical expectation of the reference parameter. It is noted that Eq. (6) above can be written in the form of a polynomial approximation depending on the magnitude of its parameter $z$. For example, given that $t=z/3.5$ for $-3.75\le z\le 3.75$ then,

$$I_0(z)=\begin{cases}\frac{0.39894228+0.01328592t^{-1}+0.00225319t^{-2}-0.00157565t^{-3}+0.00916281t^{-4}-0.02057706t^{-5}+0.02665537t^{-6}-0.0164733t^{-7}+0.00392377t^{-8}}{\sqrt{z}e^{-z}} & \text{for } -3.75\le z\le 3.75 \\ 1+3.5156229t^2+3.0899424t^4+1.2067492t^6+0.2659732t^8+0.0360768t^{10}+0.0045813t^{12} \quad \text{for } -3.75\le z\le 3.75 & \text{for } 3.75\le z\le\infty\end{cases}$$

The conditional probability density function of $Y$, $f(y/x)$, given that the peak value of $X$ has occurred can be obtained by dividing Eq. (5) by $f_x(x)$ in Eq. (4) so that

$$f(\overline{y}|x)=\frac{y}{m_{0y}(1-\rho_{xy}^2)}e^{\{(-1/(1-\rho_{xy}^2))\times(\rho_{xy}^2x^2/2m_{0x})+(y^2/2m_{0y})\}}xI_0\left(\frac{\rho_{xy}xy}{(1-\rho_{xy}^2)\sqrt{m_{0x}m_{0y}}}\right) \tag{11}$$

Therefore, the conditional complementary cumulative probability distribution of $y$ exceeding $\bar{y}$ with a given extreme value of $x$ can be written as

$$p(Y|X > \bar{y}) = \int_{\bar{y}}^{\infty} \frac{y}{m_{0y}(1-\rho_{xy}^2)} e^{\{\}\left\{(-1/(1-\rho_{xy}^2)) \times ((\rho_{xy}^2 x^2/2m_{0x}) + (y^2/2m_{0y}))\right\}} \times I_0\left(\frac{\rho_{xy} xy}{(1-\rho_{xy}^2)\sqrt{m_{0x} m_{0y}}}\right) dy \tag{12}$$

The most extreme value $\bar{y}$ of the random variable $y$ associated with the given extreme value of the random variable $x$ may be obtained iteratively in conjunction with Eq. (12) from

$$p(Y|X > \bar{y}) \equiv \frac{1}{N} \tag{13}$$

And for the case the design extreme corresponding value $\bar{y}$

$$p(Y|X > \hat{y}) \equiv \frac{\alpha}{N} \tag{14}$$

$$\text{for} \quad N = 3600 \times n \times T \tag{15}$$

and $n$ is given by

$$n = \frac{1}{2\pi}\sqrt{\frac{m_{2x}}{m_{0x}}} \tag{16}$$

where $m_{0x}$ and $m_{2x}$ are the zero- and second-order moments of the spectrum of the random process $x$; $N$ is the total number of responses in time $T$ for a given sea state and $n$ is the number of responses per unit time; $\alpha$ is a risk parameter based on extreme value statistics methodology proposed by Ochi (1990). Note that the most probable extreme value of a random variable is the value for which its extreme value probability density function peaks.

#### 3.1.2. Long-term load combination

The long-term load combination analysis is made up of a large number of short-term analyses that take into account various sea states and different wave headings. In this formulation, the probabilities of the sea state and different wave headings are considered. The long-term conditional complementary cumulative probability of exceeding extreme wave-induced load $\bar{y}$ in association with a given principal extreme load $\bar{x}$ is given below as

$$P(y|x > \bar{y}) = \sum_{ij} \frac{\sum_k nP_{ij}P_k p(y|x > \bar{y})}{\sum_{ij}\sum_k nP_{ij}P_k} \tag{17}$$

where $P(y|x > \bar{y})$is obtained from Eq. (12), $n$ is the frequency of the responses for a given short-term analysis, $P_{ij}$ is the joint probability of the wave height and wave period and $P_k$ is the probability of the wave heading. The total number of responses $N_L$ in a lifetime $L_t$ is given as

$$N_L = L_t \times 365.25 \times 24 \times 3600 \times \sum_{ij}\sum_{k} n_{ij} P_{ij} P_k \tag{18}$$

The most probable extreme value of the random variable $y$ associated with an extreme value of the random variable $x$ may be obtained by

$$P(y|x > \bar{y}) = \frac{1}{N_L} \tag{19}$$

and for the most probable design extreme value of the random variable $y$ associated with the random variable $x$

$$P(y|x > \hat{y}) = \frac{\alpha}{N_L} \tag{20}$$

Eqs. (19) and (20) can be solved iteratively in conjunction with Eq. (17) in the given analysis. The mathematical formulation above is used to determine the elements of a matrix of design extreme global wave loads both for the short-term and long-term analyses as shown below:

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} & \cdots & \cdots & f_{16} \\ f_{21} & f_{22} & f_{23} & \cdots & \cdots & f_{26} \\ \vdots & & & & & \\ \vdots & \cdots & & & & \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{61} & f_{62} & f_{63} & \cdots & \cdots & f_{66} \end{bmatrix} \tag{21}$$

$f_{ii}$ is the individual principal extreme load in the $i$th mode obtained through a typical short-term or long-term spectral analysis by taking $\rho_{xy}=0$, and $f_{ij}$ is the extreme load of mode $j$ associated with the given principal extreme load $f_{ii}$ when $i$ is not equal to $j$.

### 3.2. Key numerical results

In order to test this methodology, Alfred Mohammed et al. (2012) performed a case study for a typical 10,000 TEU containership with principal particulars given in Table 1.

A three-dimensional source distribution Green functions method developed by Chan (1992) was employed. The wetted hull surface of the containership was modelled by 1896 panels to predict the ship motions and dynamic wave loads (see Fig. 1a). Response amplitude operators (RAOs) for the motions and global wave-induced loads were obtained for a frame section in the midship region of the vessel (see Fig. 1b–e). These RAOs together with given sea statistics in terms of sea spectra, vessel headings and speed were used in conjunction with the cross-spectral probabilistic methodology outlined above to predict the most probable extreme global wave-induced loads and their associated global wave-induced loads in other modes.

The predicted global wave-induced loads were used to populate the matrix in Eq. (21). In a design situation, it would be required to select a particular load combination in which the principal load $f_{ii}$ for that combination pre-dominates the loads in other modes $f_{ij}$ and is therefore most likely to cause failure. For example, the longitudinal strength of a hull girder is usually a measure of its capacity to withstand mostly vertical bending moment $f_{55}$.

Results obtained for a section amidships are shown in Tables 2 and 3 where Table 3 are results from a similar analysis but using a methodology based on Hamilton (1993). Based on IACS (2000) Recommendation No. 34 on "Standard Wave Data", the Bretschneider two-parameter wave spectrum in conjunction with the Wave Scatter Diagram for Area 16 from the Global Wave Statistics, which represents the worst sea conditions for the North Atlantic, was used in the analyses.

Spearman's rank correlation $\rho_s$ test was used to investigate the extent to which the results obtained by the cross-spectral probabilistic method and the results obtained by the cross-spectral Hamilton's approach vary together (Alfred Mohammed et al., 2012). This covariance between the two methods and the results from Spearman's correlation test shown in Table 4 demonstrate that the design extreme wave-induced global wave loads predicted by both methods are well correlated. This simply means that though their corresponding terms might not be equal, they have similar variation in magnitude. Fig. 2 illustrates this point.

**Table 1**
Containership principal particulars.

| | |
|---|---|
| Length overall | 352.25 m |
| Length BP | 336.40 m |
| Breadth (moulded) | 42.80 m |
| Depth (moulded to upper deck) | 24.10 m |
| Mean draught | 14.35 m |
| Service speed | 23.90 knots |

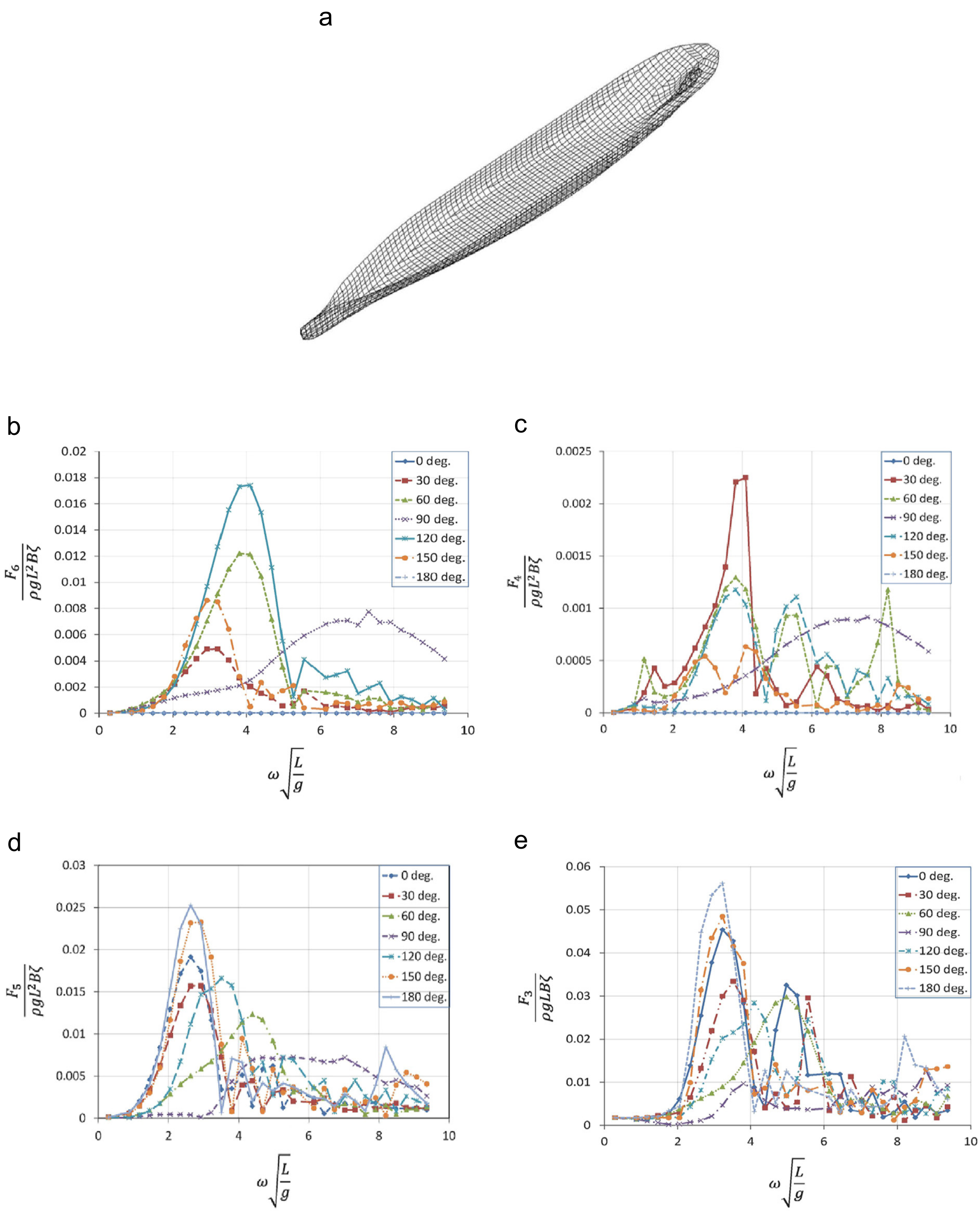


**Fig. 1.** Illustration of loads on a section amidships of a 10,000 TEU containership: (a) 1896 panels model, (b) horizontal bending moment RAOs, (c) torsional bending moment RAOs, (d) horizontal bending moment RAOs, and (e) vertical shear force RAOs.

From Fig. 2, it can be seen that the cross-spectral Hamilton's method predicted slightly higher load combinations than the cross-spectral probabilistic approach. Also, Tables 2 and 3 show that the principal design extreme global wave load is always greater than its non-principal counterpart associated with other principal design extreme global wave loads.

The cross-spectral probabilistic methodology is suitable for the reliability assessment procedure as it addresses some of the uncertainties associated with modelling some of the parameters involved in the determination of the loads imposed on the hull structure of a vessel. These parameters can also be easily studied in order to determine their importance in the scheme of the load modelling. The cross-spectral probabilistic methodology can be extended via higher order spectral analysis to study non-linear loading. It can also be applied to combine loads that are not necessarily in the same dimension and hence cannot be combined

**Table 2**
Global wave loads combinations (C1–C5) amidships for short-term analysis using the cross-spectral probabilistic method.

| | $F_1$(Nm) × $10^7$ | $F_2$(Nm) × $10^7$ | $F_3$(Nm) × $10^7$ | $F_4$(Nm) × $10^8$ | $F_5$(Nm) × $10^9$ | $F_6$(Nm) × $10^9$ |
|---|---|---|---|---|---|---|
| $C_1$ | 6.050 | 1.017 | 1.037 | 1.835 | 1.869 | 1.660 |
| $C_2$ | 3.377 | 5.546 | 2.771 | 5.293 | 4.508 | 4.763 |
| $C_3$ | 3.072 | 3.343 | 7.247 | 2.241 | 1.100 | 1.884 |
| $C_4$ | 1.767 | 3.965 | 4.648 | 6.295 | 7.865 | 5.368 |
| $C_5$ | 4.800 | 3.278 | 6.805 | 2.769 | 1.174 | 4.407 |
| $C_6$ | 4.906 | 3.892 | 3.387 | 6.042 | 7.040 | 6.940 |

**Table 3**
Global wave loads combinations (C1–C5) amidships for short-term analysis using the cross-spectral Hamilton's method.

| | $F_1$ (Nm) × $10^7$ | $F_2$ (Nm) × $10^7$ | $F_3$ (Nm) × $10^7$ | $F_4$ (Nm) × $10^8$ | $F_5$ (Nm) × $10^9$ | $F_6$ (Nm) × $10^9$ |
|---|---|---|---|---|---|---|
| $C_1$ | 6.050 | 4.780 | 4.862 | 5.122 | 7.682 | 6.080 |
| $C_2$ | 4.996 | 5.546 | 6.568 | 5.328 | 9.416 | 5.011 |
| $C_3$ | 5.379 | 2.537 | 7.247 | 4.715 | 8.371 | 5.172 |
| $C_4$ | 5.702 | 4.028 | 6.647 | 6.295 | 7.548 | 3.121 |
| $C_5$ | 6.027 | 4.185 | 4.126 | 5.371 | 1.174 | 3.992 |
| $C_6$ | 5.579 | 2.270 | 6.558 | 5.553 | 7.494 | 6.940 |

**Table 4**
Correlations between the design extreme wave-induced global loads combinations for both the cross-spectral probabilistic and Hamilton's methods.

| Forces/moments rows compared | $\rho_s$ (short-term analysis) |
|---|---|
| $C_1$ | 0.829 |
| $C_2$ | 0.943 |
| $C_3$ | 0.468 |
| $C_4$ | 0.886 |
| $C_5$ | 1.000 |
| $C_6$ | 0.771 |

by other existing methods. The idea of the use of a stochastic and probabilistic analysis in the combination of wave-induced loads also provides a platform for the inclusion of other forms of uncertainties that may be present not only at the load modelling level but also load effects analysis. The global wave load combinations that were predicted in the above analysis can be applied on an FE structural analysis to study the loads effects in the theme of the load combination analysis.

## 4. Uncertainties in seakeeping and implementation in routeing systems

The assessment of seakeeping events, of ship's structural integrity and calculations of added resistance and powering in waves are the necessary ingredients of decision support/routeing systems, which may be employed for the evaluation of the appropriate route and ship speed. In the deterministic seakeeping problem the above calculations and assessments are based on specific ship-inherent and environmental data and assumptions. However, in realistic conditions, each parameter of the seakeeping problem, even sensitive to the seakeeping ship-inherent data, such as the metacentric height and radius of gyration, is related to a degree of uncertainty (epistemic uncertainties), whereas other parameters, such as environmental data (wind, currents, and waves), are inherently random (and of aleatory uncertainty). The above uncertain parameters and random variables define a complex probabilistic problem, the solution of which is, in addition to its complexity, very time consuming for onboard decision applications.

In this paper we outline relevant research work conducted within the EU funded, FP6 project ADOPT (2005–2008) of the European Commission and thereafter. The methodology of an onboard decision support route optimisation system for the evaluation of the seakeeping parameters and the conditions affecting the ship's navigation is introduced in the following sections. Having such a system installed onboard, alternative navigational conditions may be evaluated online and the optimum route control option can be given to the master for the minimisation of the emerging risk and the minimisation of fuel cost within an acceptable route time frame. The module of the system for the estimation of the probabilities of exceedance for various seakeeping events is herein elaborated, and results of its application to the operation of a sample ship are presented and briefly discussed. It should be noted that the present paper does not address uncertainties related to the statistical estimation of extreme roll responses of ships in irregular seas. The reader may refer to a recent paper by Kim and Troesch (2013) and earlier ones by Atua and Ayyub (1997) and Ayyub et al. (2000, 2006) elaborating on the uncertainties in the probabilistic assessment of ship's dynamic stability in waves.

### 4.1. Seakeeping assessment with uncertainties

Seakeeping hazards can be formulated with a limit state function $g(X)$, where $X=(X_1, X_2,\ldots,X_N)$ is a vector of random variables and takes negative values when a hazard occurs. Therefore the probability is calculated for the cases where:

$$g(X_1, X_2, \ldots, X_N) < 0 \tag{22}$$

The presented limit state $g$-functions are herein mainly functions of ship responses in waves. By having the function $S$ of the employed ship motion model, the ship responses $Y$, the wave and loading parameters $X$ and some given response control parameters $C$, the $g$-function can be defined as:

$$Y = S(X|C) \tag{23}$$

$$g = g(X, Y|C) \tag{24}$$

In realistic conditions each parameter of the seakeeping problem, even sensitive to seakeeping ship-inherent data such as the metacentric height and radius of gyration, is related to a degree of uncertainty, whereas other parameters, such as the environmental data, are inherently random. Their uncertainty, however, is not of the same importance for the calculations of the ship's responses, as may be concluded by exploration of the response space.

### 4.2. Seakeeping hazards

In the development of the seakeeping module of the ADOPT decision support system, five limit states (hazards) were implemented, namely the vertical acceleration at the bow, the total acceleration at the bridge, the bow slamming, the propeller racing, and the deck immersion (green water). Hazards are defined either as excessive acceleration (exceeding threshold values) or high number of occurrences of seakeeping events. They are calculated for several locations along the ship, which may be readily modified in a straight-forward method. Other hazards (e.g. bending moment) may be also added in the presently implemented system.

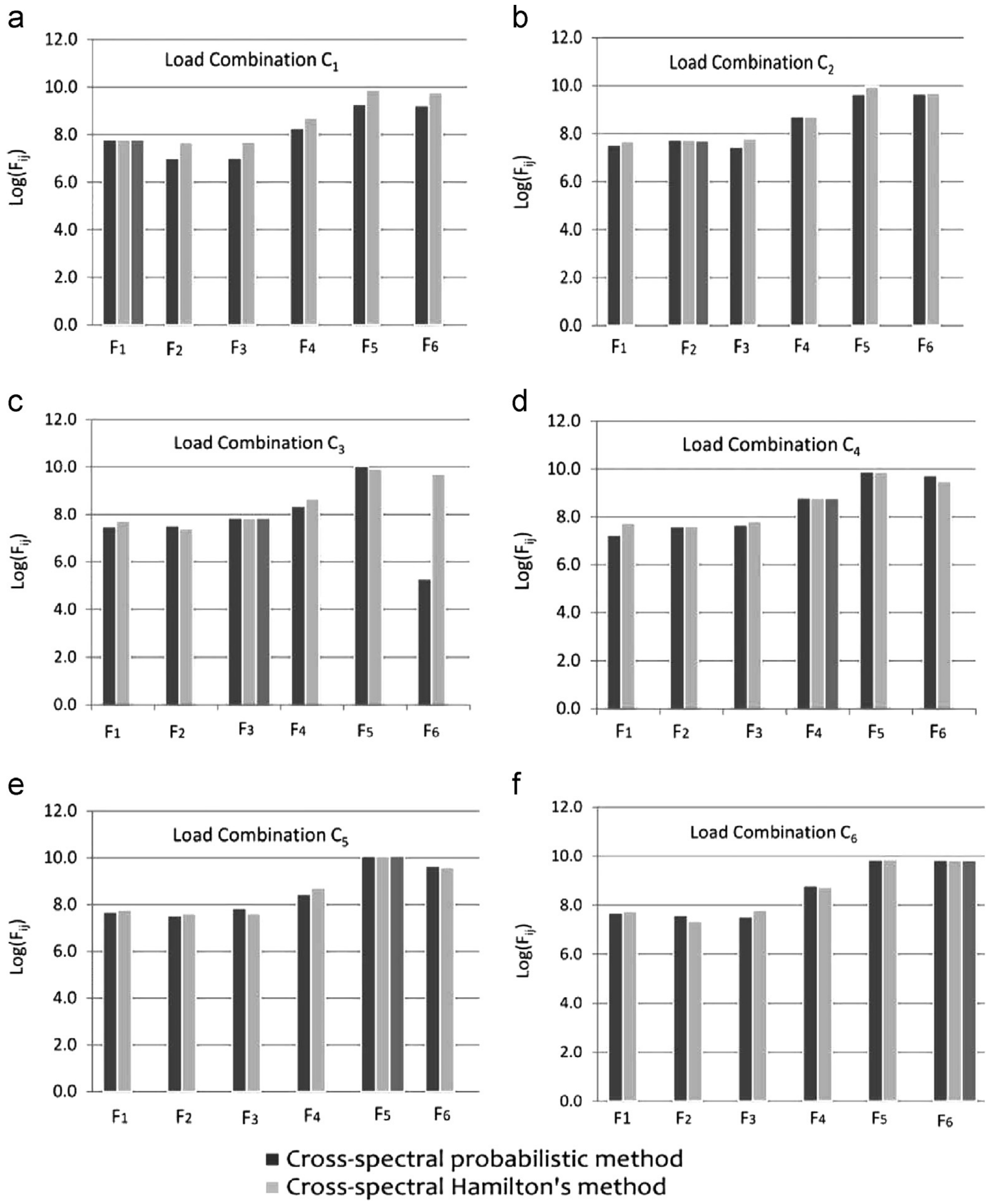


**Fig. 2.** Design extreme global wave loads combinations amidships for short-term analysis.

Formulation of limit states was used for the evaluation of the mean up-crossing rate of some variables. For the Gaussian, zero-mean, and narrow-band processes, the mean up-crossing rate $\nu^+$ of a level $\alpha$ can be approached by

$$\nu^+ = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\exp\left(-\frac{\alpha^2}{2m_0}\right) \tag{25}$$

where $m_0$ and $m_2$ are the zero- and second-order moment of the variable's spectrum $S_R$ in consideration. For linear ship responses, the $S_R$ is calculated from the transformation of the wave spectrum by means of the response operator $H$, both functions of frequency $\omega$

$$S_R(\omega) = |H(\omega)|^2 S(\omega) \tag{26}$$

### 4.3. Threshold values

The hazards are defined through a set of characteristic threshold values for the involved variables. Suppose the hazard of the frequent propeller racing that occurs during severe pitching and subsequent propeller emergences. Such an event is undesirable for the propulsion system. So, the capacity of the propulsion system and its tolerance of the racings are analysed independently of the ship motions, and then the threshold value for the racing rate is determined and correlated consequences are attributed. If the frequency of propeller racing is higher than the determined threshold value, then the ship will encounter the related consequences. Apparently, for a hazard on a top level description several threshold values may be derived with each one correlated to a different level of consequences.

### 4.4. Probabilistic assessment methods

For the evaluation of the probabilities of the seakeeping hazards, the first-order reliability method (FORM), second-order reliability method (SORM), and Monte Carlo method have been employed and investigated.

The FORM method initially transforms the basic $X$-variable space of a formulated limit state function $g(X)$ into a u-space, where variables $U_i$ are independent and standard normal

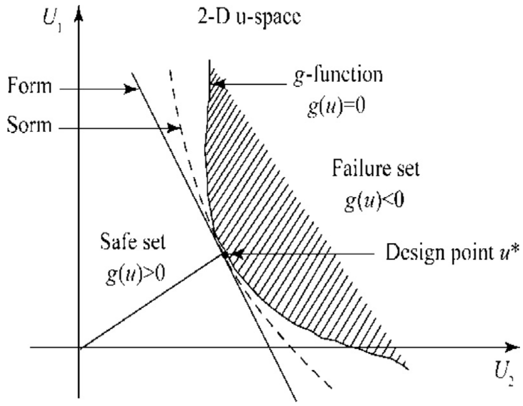


**Fig. 3.** Two-dimensional $g$-function approach with FORM.

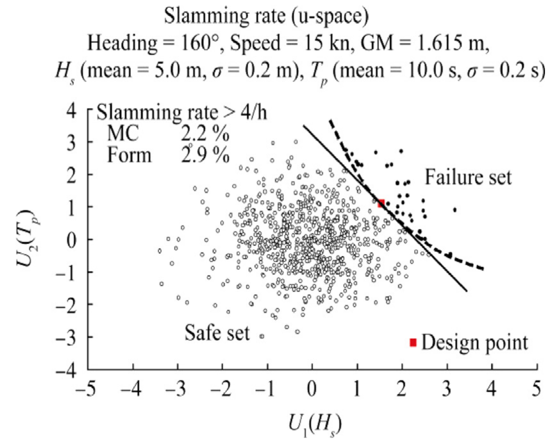


**Fig. 4.** U-space for $H_s$ and $T_p$.

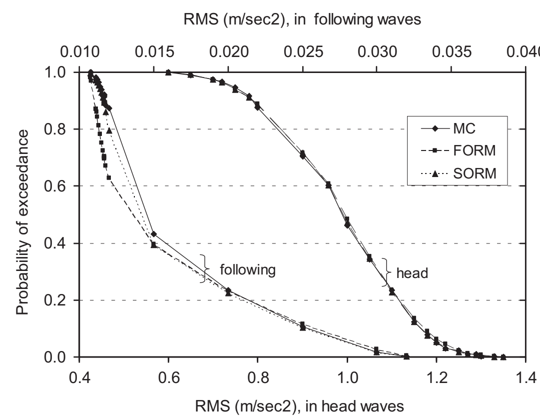


**Fig. 5.** Probability of the vertical bow acceleration.

variables. The mapping does not change the distribution of the $g$-variable and preserves all relevant probability information. In this way, the transformed $g$-function divides the u-space into safe and failure domains, Fig. 3, where $g>0$ and $g<0$ correspondingly. Then, if the $g$-function is substituted by a linear function which passes through a point $u^*$, the so-called design point, which is the point of the g-function closest to the space origin, a first-order approximation is defined, namely the FORM method. Thus, the failure probability corresponds to the sub-domain defined by the linear approximation instead of the actual $g$-function (the shaded set in Fig. 3). Applying the same concept, but implementing a second-order approximation, the SORM (second-order reliability) method is defined. Obviously if the limit surface $g$ of a hazard is not strongly non-linear, then the approximation defined by FORM and corresponding probabilities could be satisfactory in view of the accuracy for the set problem.

The Monte Carlo method that is based on sampling of the evaluated function proves efficient for the calculation of the *central* part of the distribution. Nevertheless, for low probability events (tails of distribution) it suffers from the large number of simulations required to achieve a satisfactory level of accuracy.

In Fig. 4, the u-space mapping for the significant wave height $H_s$ (mean$=$5.0 m, $\sigma$$=$0.2 m) and the peak period $T_p$ (mean$=$ 10 s, $\sigma$ $=$0.2 s) is shown for a slamming rate$>$4 per hour of a RoPax ship, sailing with a speed of 15 knots in 160° bow waves. The straight line is the linear approximation of the $g$-function according to FORM (MC: Monte Carlo simulation).

Fig. 5 presents the probability of exceedance of an acceleration level at the study RoPax's bow as determined with three different probability assessment methods, namely Monte Carlo, FORM and SORM. All the methods converge for the lower probability levels, whereas differences are notable in the left part of the diagram, namely for the higher probabilities, and in the case of the following waves. However, in the range of lower probabilities, where the main interest for the DSS is defined, the FORM behaviour has been found rather satisfactory. Based on such findings it can be concluded that when high probabilities are encountered then estimations by FORM should be verified by use of an alternative Monte Carlo simulation, which is expected to converge rapidly with a reasonable number of trials as the probability level is already considerably high.

### 4.5. Practical application

Within the EU funded ADOPT (2008) project the Ship Design Laboratory of NTUA interfaced the in-house developed seakeeping, 3D panel code NEWDRIFT (Papanikolaou and Schellin, 1992) and the probabilistic assessment tool PROBAN of DNV (Spanos et al., 2008; DNV 2003). From the analysis it was concluded that the ADOPT decision support system can be applied to both ship's operational assessment and the design procedure. More specifically:

- *In the operational mode* application, the parameters relevant to the ship's loading condition can be determined at the beginning of the voyage. In this mode random variables are the environmental parameters. The speed and the heading to the wave constitute the control parameters. The onboard-online probabilistic evaluation is enabled by short execution times of the seakeeping module of ADOPT.
- *In the design mode*, all the variables are considered uncertain. This mode can be useful at the design stage for the risk assessment during the ship's life. The requested time for the probabilistic calculation does not allow this mode to be feasible on an onboard route optimisation system.

A fast computational performance in order to achieve practical application times onboard is a prerequisite for the developed computer-based probabilistic approach. Although the computational time to complete a full set of calculations and evaluations strongly depends on the employed computer machine, the time recorded and provided herein enables a representative view of the current performance achieved in a laboratory environment.

With reference to a single PC, Intel Core2 CPU 6600 @ 2.40 GHz, 2 GB RAM, and for a dense hull representation (2 × 500 panels), the computational times are:

- 35 s per limit state evaluation, when using Monte Carlo.
- 5 s per limit state evaluation, when using FORM.

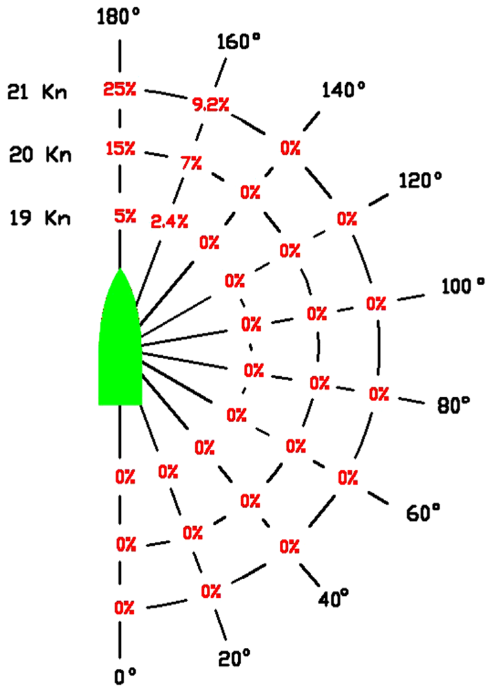


**Fig. 6.** Probabilities for propeller emergence rate > 1/min.

An evaluation of five limit states takes about 12.5 min for the calculations of the probabilities for 30 alternative sailing conditions within a range of speed-heading combinations, as shown in Fig. 6. For this assessment the wave spectrum parameters have been assumed to be uncertain parameters. The required time for the evaluation of five limit states can be considered short enough for use of this module in an onboard route optimisation system (see, Papatzanakis et al., 2012). It should be noted that the above cited time performance values refer to a PC hardware technology that has been significantly enhanced in the last few years. Thus, the proposed concept can be even more efficiently implemented by todays IT hardware standards.

## 5. Conclusive remarks

In this paper the authors, in an attempt to stress the importance of understanding and of integrating uncertainties in the context of useful prediction tools for the assessment of ship wave-induced design loads and operations, elaborate on some indicative examples of relevant recent research work.

It is demonstrated that the mathematical formulations for the cross-spectral probabilistic method could be used within the context of uncertainty to combine global wave loads as well as combinations of global with local wave loads within the context of short-term or long-term statistical approaches. Accordingly, uncertainties associated between different methods may be related with the actual determination of the correlation functions and the corresponding relationships of the wave-induced loads. For example, Hamilton's cross-spectral method which is based on the computation of a variance that represents the distribution of an associated load when the principal load is at its peak; however, the cross-spectral probabilistic method establishes a relationship between wave-induced dynamic loads through a correlation factor together with a bi-variate Rayleigh distribution of the wave-induced load responses concerned.

As a practical application of uncertainty modelling to the assessment of ship operation, a ship specific, risk-based decision support system DSS for onboard guidance of the master with respect to the seakeeping of the ship was also introduced. Some fundamental issues and properties of this DSS have been discussed; the need for an efficient probabilistic assessment method in the core of the DSS system proves to be a prerequisite for practical onboard applications, especially if employed within routeing optimisation schemes, aiming at mitigating environmental risks and reducing fuel consumption by evaluating efficiently a large number of alternative routeing scenarios and environmental conditions.


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